Attitude control of a rigid spacecraft with two momentum wheel actuators using genetic algorithm

Xin-Sheng Ge\textsuperscript{a,b}, Li-Qun Chen\textsuperscript{b,*}

\textsuperscript{a}Basic Science Courses Department, Beijing Institute of Machinery, Beijing 100085, China
\textsuperscript{b}Shanghai University, Shanghai Institute of Applied Mathematics and Mechanics, Shanghai 200072, China

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Abstract

The control problem of the spacecraft attitude using two momentum wheel actuators is investigated. It is well-known that attitude of a rigid spacecraft can be controlled by using three momentum wheels. If one of the momentum wheels is failure, the complete spacecraft equations are not controllable. When the total angular momentum of the system is zero, the control problem of the spacecraft attitude becomes a steering problem for a drift free control system. In this paper, based on the optimal control theory, a genetic algorithm for steering a rigid spacecraft with two momentum wheels is proposed. The genetic algorithm provide the results of a numerical simulation to prove its efficiency and stability.

1. Introduction

The attitude control problem of a rigid spacecraft using angular rate of the two momentum wheels as inputs is considered. A rigid spacecraft in general is controlled by three independent actuators and it is well known that three momentum wheels can be used to accomplish arbitrary reorientation maneuvers of the spacecraft. If failure of one of the wheels occurs, then one is left with only two momentum wheels of spacecraft. The attitude control should be performed by using control inputs for underactuated spacecraft with the remaining wheels. Thus the attitude control of a spacecraft operating in an actuator failure mode is an important control problem. Attitude control of a rigid spacecraft using two control inputs has been addressed by several researchers. Controllability results for a rigid spacecraft controlled by momentum wheel actuators was proposed by Crouch [1]. Aeyels [2] and Krishnan et al. [3,4] investigated the stabilization of the angular velocity equations of a rigid spacecraft using less than three momentum wheels or gas jet actuators. Motion planning and attitude stabilization problems for a rigid spacecraft with less than three independent control inputs have been studied by Tsiotras et al. [5]. Bloch et al. [6] gave an expression for the geometric phase of the attitude of a rigid spacecraft with two momentum wheel actuators under the assumption that the total angular momentum vector of the system is zero. Walsh et al. [7] make use of Lie algebraic methods to provide controllability results and to obtain specific control constructions. Fernandes et al. [8,9] presented a variational optimal algorithm (near-optimal algorithm) to solve feasible trajectories for systems of a satellite with two rotors and coupled rigid
bodies. This method is based on Gauss–Newton iteration method in the optimal control to search for the optimal solution. Ge et al. [10] studied an optimal control of stretching process of solar arrays on spacecraft using genetic algorithm.

This present paper is devoted to study the problem of finding a solution for a rigid spacecraft with two momentum wheel actuators using a genetic algorithm, instead of traditional optimal method—Newton iteration method. A search technique that has become established in the field of optimization as extremely technique is the genetic algorithm (GA) [11]. GA is search algorithms based on the mechanics of natural genetics, they use operations found in natural genetics to guide their trek through the search space. GA search through large spaces quickly, requiring only objective function value information to guide their search, an inviting characteristic since the majority of commonly used search techniques require derivative information, continuity of the search space, or complete knowledge of the objective function to guide their search. Furthermore, because of the processing leverage associated with GA, they take a more global view of the search space than many traditional methods. In this paper, we take advantage of multibody dynamics to derive the dynamics equations for a spacecraft with two momentum wheel actuators. Then, we provide the GA of optimal control, and set up the control objective function of the system, define the genotype of chromosome and the function of fitness, and design the corresponding genetic operation. Finally, the last section discusses the numerical simulation, which is shown that this method can be computed to effect an arbitrary reorientation of a rigid spacecraft.

2. Modeling the spacecraft

Consider the rotational dynamics of a rigid spacecraft controlled by two momentum wheels. We use the Cardan angle convention for parameterizing the orientation of the rigid spacecraft. Assume an inertial \( XYZ \) coordinate frame, let \( xyz \) be a coordinate frame aligned with principal axes of the spacecraft with origin at the center of mass of the spacecraft. If the two frames are initially coincident, a series of three rotations about the body axes performed in the proper sequence is sufficient to allow the spacecraft to reach any orientation. The corresponding rotation matrix is denoted as \( R (\theta, \psi, \phi) \), where \( \theta, \psi \) and \( \phi \) are Carden angle [12]. The mass of the spacecraft, wheel 1 and wheel 2 are denoted as \( m_0, m_1 \) and \( m_2 \), respectively. The inertial tensors of the spacecraft are denoted as \( I_0, I_1 \) and \( I_2 \). Assume \( \rho_0, \rho_1 \) and \( \rho_2 \) denoted by the position vectors of the center of mass of the spacecraft, wheel 1 and 2, respectively, with respect to the center of mass of the whole system. Thus, from the location of the wheels

\[
\begin{align*}
\rho_1 &= \rho_0 + d_1 b_1, \\
\rho_2 &= \rho_0 + d_2 b_2,
\end{align*}
\]

where \( b_1, b_2 \) are defined unit vectors of wheel 1 and 2 spinning about axes fixed in the rigid spacecraft (see Fig. 1). The \( d_1, d_2 \) are constants that are the distances of wheels 1 and 2 from the center of mass of the spacecraft along the unit vectors \( b_1 \) and \( b_2 \), respectively. Further \( b_1 \) and \( b_2 \) span a two-dimensional plane that is orthogonal to a principal axis of the spacecraft. Without loss of generality, the \( b_1 \) and \( b_2 \) are assumed to be of the form

\[
\begin{align*}
b_1 &= (b_{1x}, b_{1y}, 0)^T, \\
b_2 &= (b_{2x}, b_{2y}, 0)^T.
\end{align*}
\]

By the definition of center of mass, we can obtain expressions for \( \rho_0, \rho_1 \) and \( \rho_2 \). The total angular momentum vector of the system is given, in the spacecraft
body frame, by
\[
R(\theta, \psi, \phi)H = J\omega + \sum_{i=1}^{2} I_i (\omega + b_i \dot{\theta}_i). \tag{3}
\]
where \( \theta_i \) (i = 1, 2) is the angles of rotation of wheels 1 and 2 about the axis defined by \( b_i \) (i = 1, 2) respectively, the \( j_i \) (i = 1, 2) is the moment of inertia of wheel \( B_i \) (i = 1, 2) about the axis defined by \( b_i \).

Suppose the angular momentum vector \( H \) of the system is zero. Eq. (3) can be expressed in the form
\[
\omega = -\left( J + \sum_{i=1}^{2} I_i \right)^{-1} \sum_{i=1}^{2} b_i \dot{\theta}_i. \tag{4}
\]

Note \( \omega_x, \omega_y, \) and \( \omega_z \) are the principal axis components of the absolute angular velocity vector \( \omega \) of the spacecraft. Then we have
\[
\omega = L\dot{q} = \begin{bmatrix} \cos \psi \cos \phi & \sin \phi & 0 \\ -\cos \psi \sin \phi & \cos \phi & 0 \\ \sin \psi & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix}. \tag{5}
\]

Substituting the expressions (5) into Eq. (4), we obtain
\[
\dot{q} = -L^{-1} \left( J + \sum_{i=1}^{2} I_i \right)^{-1} \sum_{i=1}^{2} b_i \dot{\theta}_i. \tag{6}
\]

Note in (6) the conservation of angular momentum is nonintegrable. It is now well understood in the literature that system with nonintegrable velocity constraints or nonholonomic constraint.

### 3. The optimal control

The configuration \( q = (\theta, \psi, \phi)^T \) of the system is defined as a state variable. Consider the angles velocity \( \dot{\theta}_i \) (i = 1, 2) of rotation of wheels 1 and 2 about the axis defined by \( b_i \) (i = 1, 2) as control input variable, noted as \( u = (\dot{\theta}_1, \dot{\theta}_2)^T \). Then, the equation of motion of the controlled system takes the form
\[
\dot{q} = B(q)u,
\]
where parameter \( \lambda \) is a penalty factor. It has been proved that the solution of problem (10) converges to the optimal solution as \( N, \lambda \to \infty \) [9]. For any \( \alpha \in R^N \), the solution of Eq. (7) at time \( t = T \), is a function of \( \alpha \) and is denoted by \( f(\alpha) \). Thus, the objective function becomes
\[
J(\alpha) = \langle x, x \rangle + \lambda \| f(x) - q_f \|^2. \tag{11}
\]
Therefore, the problem of making the objective function $J(u)$ reaches minimal by searching for the control input $u$ is converted to that of making the minimized solution of the objective function (11) searching $z$. To apply Newton’s algorithm, we need to compute the function $f(z)$ and its Jacobian. Fernandes et al. [9] presented a numerical algorithm, via the integration of two differential equations, to this problem.

### 4. Genetic algorithm of optimal control

We employ GA in the optimal control instead of traditional gradient method. In general, binary strings are used to encode the optimal parameter space in GA. When the dimensions of parameter become high and the range of parameter value becomes great, the converge speed of the algorithm will become slow. The real-coded GA [13] has the advantages of higher precise and convenience to search in large space, it is utilized in the paper. Considering the optimal control problem for a rigid spacecraft with two momentum wheel actuators, the GA is designed to obtain angular rate input rule of two momentum wheels and the optimal trajectory of spacecraft attitude motion. GA is designed as follows:

1. Chromosomes representation: utilizing the parallel searching mechanics of GA, $z$, the projection of function $u(t)$ on Fourier basis in Eq. (9), are encoded to an $N$-dimension vector composed by $z_i$ ($i = 1, 2, \ldots, N$)

\[
z = [z_1 z_2 \ldots z_N]^T.\]

2. Initialization of the population of parent: the $P$ parents, whose components are normal Gauss random variables, are generated randomly.

3. The selection of fitness function: we define the fitness score function as

\[
g(z) = 1/J(z),\]

where $J(z)$ is the target function defined in Eq. (11), $z$ is the chromosomes with small target function value has high fitness score.

4. Selection: the fitness score $g(z_i)$ ($i = 1, 2, \ldots, P$) of each chromosome is calculated according to Eq. (13), the survival probability of $i$th chromosome is given by

\[
P_i = \frac{g(z_i)}{\sum_{i=1}^{P} g(z_i)}.\]

This process is accomplished by using roulette wheel selection.

5. Crossover: a simple one-point crossover is employed. According to the crossover probability $P_c$, a splice point is determined uniformly at random. The genetic codes following the splice point are interchanged, and offspring are generated as shown in Fig. 2

6. Mutation: some components $z_{ij}$ ($i = 1, 2, \ldots, P$, $j = 1, 2, \ldots, N$) of the chromosome $z_i$ are chosen to mute according to the mutation probability $P_m$, and a Gauss random variable on it

\[
z_{ij} = z_{ij} + \delta_j,\]

where $\delta_j$ is Gauss random variable. The steps (4)–(6) is repeated, the fitness scores of population become higher and the optimal solution can be obtained in the end.

### 5. Simulation examples

We illustrate the results of the paper using an example. Consider a rigid spacecraft with no momentum wheels about the third principal axis and two wheels are applied about the other two principal axes. Therefore the vectors $b_1$ and $b_2$ are given by $b_1 = (1, 0, 0)^T$, $b_2 = (0, 1, 0)^T$. In the simulation, we use the mass and geometry parameters of the spacecraft with two momentum wheel actuators as follows [3]:

\[
d_1 = d_2 = 0.2 \text{ m},\]

\[
j_1 = j_2 = 0.5 \text{ kg m}^2,\]

\[
m_0 = 500 \text{ kg}, \quad m_1 = m_2 = 5 \text{ kg},\]

\[
I_0 = \text{diag}(86.215, 85.07, 113.565) \text{ kg m}^2,\]
The control parameters of the GA are selected as: the dimension of the population of parents is \( P = 32 \), the length of chromosomes is \( N = 10 \), the probability of crossover is \( P_c = 0.9 \), the probability of mutation is \( P_m = 0.1 \). The number of evolution generations generation is \( R = 6000 \). For our simulation, we have used 10 terms of the Fourier basis, the time is \( t = 5 \) s. These basis elements are

\[
e_1 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} \sin t \\ 0 \end{bmatrix},
\]

\[
e_3 = \begin{bmatrix} \cos t \\ 0 \end{bmatrix}, \quad e_4 = \begin{bmatrix} \sin 2t \\ 0 \end{bmatrix}, \quad e_5 = \begin{bmatrix} \cos 2t \\ 0 \end{bmatrix}
\]

and the remaining terms are obtained by permuting the rows of the above elements.

We choose Carden angle to parameterize the orientation of the spacecraft and the initial and final configurations are

\[
q_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad q_f = \begin{bmatrix} 0 \\ 0 \\ \pi/6 \end{bmatrix},
\]

where the final configuration is a rotation from the initial configuration about the third axis without a wheel. The results of the simulation is shown in Figs. 3 and 4, where Fig. 3 shows the optimal input for the angles velocity of rotation of two momentum wheels about the principal axes. Fig. 4 shows the \( \theta, \psi \) and \( \phi \)
components of the optimal trajectory of a rigid spacecraft linking $q_0$ to $q_f$. In these figures, the solid curve is the calculation result obtained using a GA, the dashed curve is the calculation result obtained using Gauss–Newton iteration method. The error norm in this example is of the order of $10^{-3}$. The cost of the control approximating after 6000 iteration, $\|x\| = 6.1624$.

6. Conclusions

(1) It is a new and useful attempt to introduce the GA into the optimal control of nonlinear systems. The simulation calculations show that GA is an effective method to solve the optimal control of the attitude motion of a rigid spacecraft with two momentum wheel actuators.

(2) In the GA, theoretically, binary string is more reasonable than real representations. But for real-valued numerical optimization problem considered in this paper, real representations are a best selection, because they are more comprehensible, more precise and conducive to faster execution.

(3) Differential and continuity condition is not required in the GA. Compared with traditional gradient method, it can be applied to a larger range, and is global optimal algorithm.

(4) The GA of optimal control for the spacecraft with two momentum wheel actuators is proposed in this paper, and it also supplies a new way to solve other optimal control of nonlinear system.

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References